MAC 2311 Additional Related Rates Problems

1. Let A be the area of a square with sides that are length x and assume x varies with time. At a certain instant in time, the sides are 3 feet long and growing at a rate of 2 ft/min. How fast is the area of the square growing at that instant in time? (ANS: 12 sq ft/min)

2. Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of 6 sq. miles/hour. How fast is the radius of the spill increasing when the radius is 9 miles? (ANS: $\frac{1}{3\pi}$ mph)

3. A 13-foot long ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft/sec., how fast will the base be moving away from the wall when the top of the ladder is 5 feet above the ground? (ANS: 5/6 feet/sec)

4. Sand pouring from a chute forms a conical pile whose height is always equal to the diameter. If the height increases at 5 ft/min, at what rate is the sand pouring from the chute when the pile is 10 feet high? (ANS: 125π cubic feet per minute)

5. A weather balloon rising from the ground at 140 ft/min is tracked by a rangefinder located 500 feet from the point of liftoff. Find the rate at which the angle from the range finder to the balloon and the straight-line distance to the balloon are changing when the balloon is 500 feet above the ground. (ANS: $70\sqrt{2}$ feet/minute and 0.14 radians per minute)

6. A spherical balloon is to be deflated so that its radius decreases at a constant rate of 15 cm/min. At what rate must the air be released from the balloon when the radius is 9 cm? (ANS: -4860π cm^3/min)

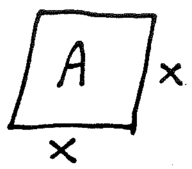
7. A conical water tank that is placed vertex down has a radius of 10 feet at the top and is 24 feet high. If water flows into the tank at a rate of 20 cubic feet / minute, how fast is the depth of the water increasing when the water is 16 feet deep? (ANS: $\frac{9}{20\pi}$ ft/min)

8. How fast does the water level drop when a cylindrical tank is drained at a rate of 3 liters/sec? (ANS: $\frac{3}{\pi r^2}$ liters/second)

9. A highway patrol plane flies 1 mile above a straight road at a steady ground speed of 120 mph. The pilot sees an oncoming car and, with radar, determines that the line-of-sight distance from the plane to the car is 1.5 miles, decreasing at a rate of 136 mph. What is the car's speed along the highway? (ANS: 62.5 mph)

Related Rate Problems

1



know: $\frac{dx}{dt} = 2$ ft/min

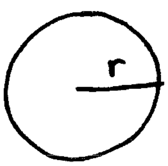
need: $\frac{dA}{dt}$ when $x = 3$

equation

$A = x^2$

$\frac{dA}{dt} = 2x\left[\frac{dx}{dt}\right]$; $\frac{dA}{dt} = 2(3)(2) = 12$ ft²/min.

2



know: $\frac{dA}{dt} = 6$ mi²/hr

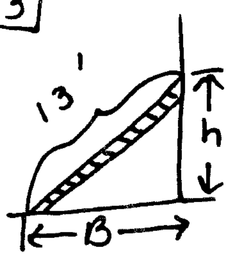
need: $\frac{dr}{dt}$ when $r = 9$ miles

equation

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\left[\frac{dr}{dt}\right]$; $6 = 2\pi(9)\left[\frac{dr}{dt}\right]$; $\left[\frac{dr}{dt}\right] = \frac{6}{18\pi} = \frac{1}{3\pi}$ mph

3



know: $\frac{dh}{dt} = 2$ ft/sec

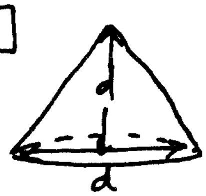
need: $\frac{dB}{dt}$ when $h = 5$, $B = 12$

equation

$B^2 + h^2 = 169$

$2B\left[\frac{dB}{dt}\right] + 2h\left[\frac{dh}{dt}\right] = 0$; $2(12)\left[\frac{dB}{dt}\right] + 2(5)(2) = 0$; $\left[\frac{dB}{dt}\right] = \frac{5}{6}$ ft/sec.

4



know: $\frac{dh}{dt} = 5$ ft/min

need: $\frac{dV}{dt}$ when $h = 10$

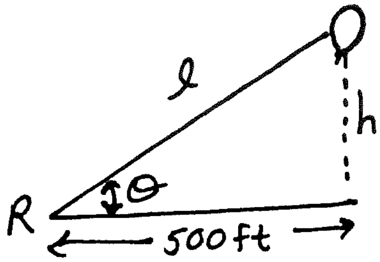
equation

$V = \frac{1}{3}\pi r^2 h$ $\quad r = \frac{1}{2}h \Rightarrow V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h \Rightarrow V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2\left[\frac{dh}{dt}\right]$; $\frac{dV}{dt} = \frac{1}{4}\pi(10)^2(5)$

$\frac{dV}{dt} = 125\pi$ ft³/min

5



have

$\frac{dh}{dt} = 140 \text{ ft/min}$

need

$\frac{d\theta}{dt}$ and $\frac{dl}{dt}$ when $h = 500$ ft

equation

$500^2 + h^2 = l^2$

$0 + 2h\left[\frac{dh}{dt}\right] = 2l\left[\frac{dl}{dt}\right]$; $2(500)(140) = 2(500\sqrt{2})\left[\frac{dl}{dt}\right]$

$500^2 + 500^2 = l^2$

$l = 500\sqrt{2}$

$140{,}000 = 1000\sqrt{2}\left[\frac{dl}{dt}\right]$

$70\sqrt{2} = \left[\frac{dl}{dt}\right]$

$\tan(\theta) = \frac{500}{500} = 1$

$\tan^{-1}(1) = \pi/4$

$\theta = \pi/4$

$\tan\theta = \frac{h}{500}$ or $\tan\theta = 1/500\, h$

$\sec^2(\theta)\left[\frac{d\theta}{dt}\right] = 1/500\left[\frac{dh}{dt}\right]$

$\sec^2(\pi/4)\left[\frac{d\theta}{dt}\right] = 1/500\,[140]$

$\left[\frac{d\theta}{dt}\right] = \frac{.28}{2} = .14$ radians/min

6

know

$\frac{dr}{dt} = 15$ cm/min

need

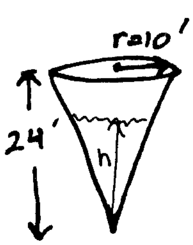
$\frac{dV}{dt}$ when $r = 9$ cm

equation

$V = 4/3\,\pi r^3$; $\left[\frac{dV}{dt}\right] = 4\pi r^2\left[\frac{dr}{dt}\right]$; $\left[\frac{dV}{dt}\right] = \pi\, 4(9)^2[15]$

$\left[\frac{dV}{dt}\right] = 4860\pi \text{ cm}^3/\text{min}$

7



know

$\frac{dV}{dt} = 20 \text{ ft}^3/\text{min}$

need

$\frac{dh}{dt}$ when $h = 16$

equation

$V = 1/3\,\pi r^2 h$ need to solve for "r" in terms of h

$\frac{r}{h} = \frac{10}{24}$

$24r = 10h$

$r = \frac{10h}{24} = \frac{5h}{12}$

$V = 1/3\,\pi\left(\frac{5h}{12}\right)^2 h$; $V = \frac{25\pi}{432} h^3$

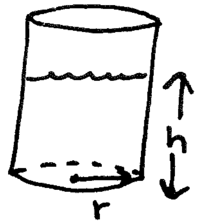
$\frac{dV}{dt} = \frac{75\pi}{432} h^2\left[\frac{dh}{dt}\right]$

$20 = \frac{75\pi}{432}[16]^2\left[\frac{dh}{dt}\right]$

$\frac{9}{20\pi} = \frac{dh}{dt}$

$\left[\frac{dh}{dt}\right] = \frac{9}{20\pi}$ ft/min

8



know

$\frac{dV}{dt} = 3$ liters/sec

need

$\frac{dh}{dt}$

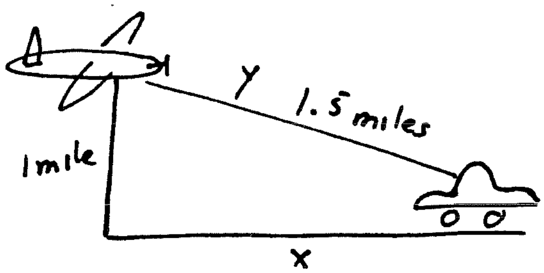
equation

$V = \pi r^2 h$ (r is a constant)

$\frac{dV}{dt} = \pi r^2 \left[\frac{dh}{dt}\right]$

$3 = \pi r^2 \left[\frac{dh}{dt}\right]$; $\left[\frac{dh}{dt}\right] = \frac{3}{\pi r^2}$ liters/second

9



$1^2 + x^2 = 1.5^2$

$x^2 = 1.5^2 - 1^2$

$x^2 = 1.25$

$x = \sqrt{1.25}$

know

$\frac{dy}{dt} = -136$ mph

need

$\frac{dx}{dt}$

equation

$1^2 + x^2 = y^2$

$0 + 2x\left[\frac{dx}{dt}\right] = 2y\left[\frac{dy}{dt}\right]$

$2\sqrt{1.25}\left[\frac{dx}{dt}\right] = 2(1.5)(-136)$

$\left[\frac{dx}{dt}\right] = \frac{-408}{2\sqrt{1.25}} \cong -182.5$ mph

the plane is going 120 mph

so the car must be going

$-182.5 + 120 = 62.5$ mph